Power in a Balanced Three-Phase System

- To find total power in a balanced system
  - Determine power in one phase
  - Multiply by three

- You can also use single-phase equivalent in power calculations
  - Power will be power for just one phase

Three-Phase Active (Average) Power

- Active power per phase = $V_\phi I_\phi$ x power factor
- Total active power = **3** $V_\phi I_\phi$ x power factor

$$P = 3V_\phi I_\phi \cos\theta$$

➢ If I_L and V_L are rms values for line current and line voltage respectively. Then for delta (Δ) connection: $V_\phi = V_L$ and $I_\phi = I_L/\sqrt{3}$. therefore:

$$P = \sqrt{3} V_L I_L \cos\theta$$

➢ For star connection (Y) : $V_\phi = V_L/\sqrt{3}$ and $I_\phi = I_L$. therefore:

$$P = \sqrt{3} V_L I_L \cos\theta$$

Three-Phase Instantaneous Power

Instantaneous Phase Voltages

$$v_{an}(t) = V_m \sin(\omega t)$$
$$v_{bn}(t) = V_m \sin(\omega t - 120°)$$
$$v_c(t) = V_m \sin(\omega t - 240°)$$

Instantaneous Phase Currents

$$i_a(t) = I_m \sin(\omega t - \theta)$$
$$i_b(t) = I_m \sin(\omega t - \theta - 120°)$$
$$i_c(t) = I_m \sin(\omega t - \theta - 240°)$$

$$v_{an}(t) = \sqrt{2}V \sin \omega t$$
$$v_{bn}(t) = \sqrt{2}V \sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V \sin(\omega t - 240^0)$$

$$i_a(t) = \sqrt{2}I \sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I \sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I \sin(\omega t - 240^0 - \theta)$$

Three-Phase Instantaneous Power

✓ Instantaneous Power

$$p(t) = v(t)i(t)$$

➢ Therefore, the instantaneous power supplied to each phase is:

$$p_a(t) = v_{an}(t)i_a(t) = 2VI\sin(\omega t)\sin(\omega t - \theta)$$

$$p_b(t) = v_{bn}(t)i_b(t) = 2VI\sin(\omega t - 120^0)\sin(\omega t - 120^0 - \theta)$$

$$p_c(t) = v_{cn}(t)i_c(t) = 2VI\sin(\omega t - 240^0)\sin(\omega t - 240^0 - \theta)$$

- Since

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

Three-Phase Instantaneous Power

- Therefore

$$p_a(t) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$p_b(t) = VI\left[\cos\theta - \cos(2\omega t - 240^0 - \theta)\right]$$

$$p_c(t) = VI\left[\cos\theta - \cos(2\omega t - 480^0 - \theta)\right]$$

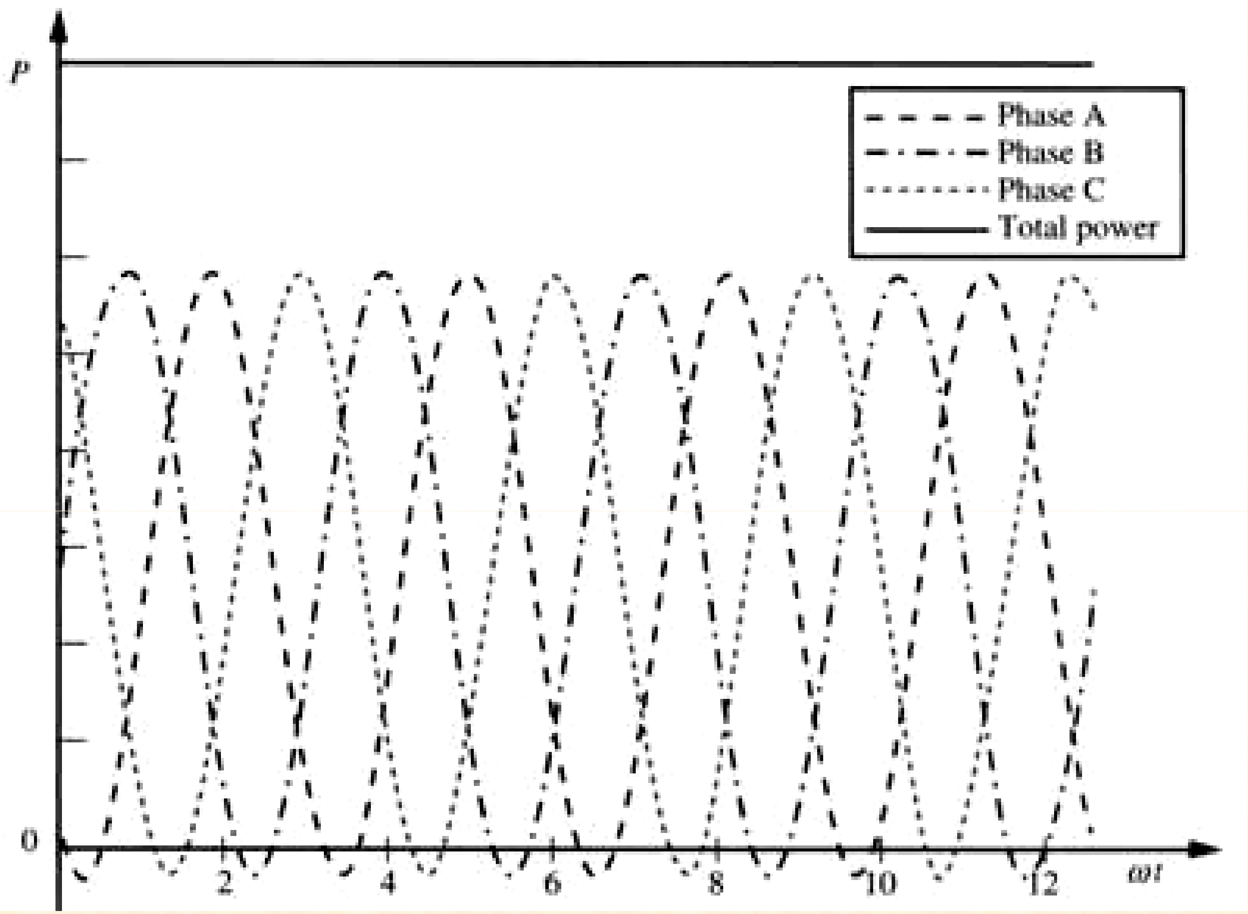
➢ The total instantaneous power

$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI\cos\theta$$

Note that: the pulsing components cancel each other because of 120^0 phase shifts.

✓ **For a balanced three phase circuit the instantaneous power is constant**

Three-Phase Instantaneous Power

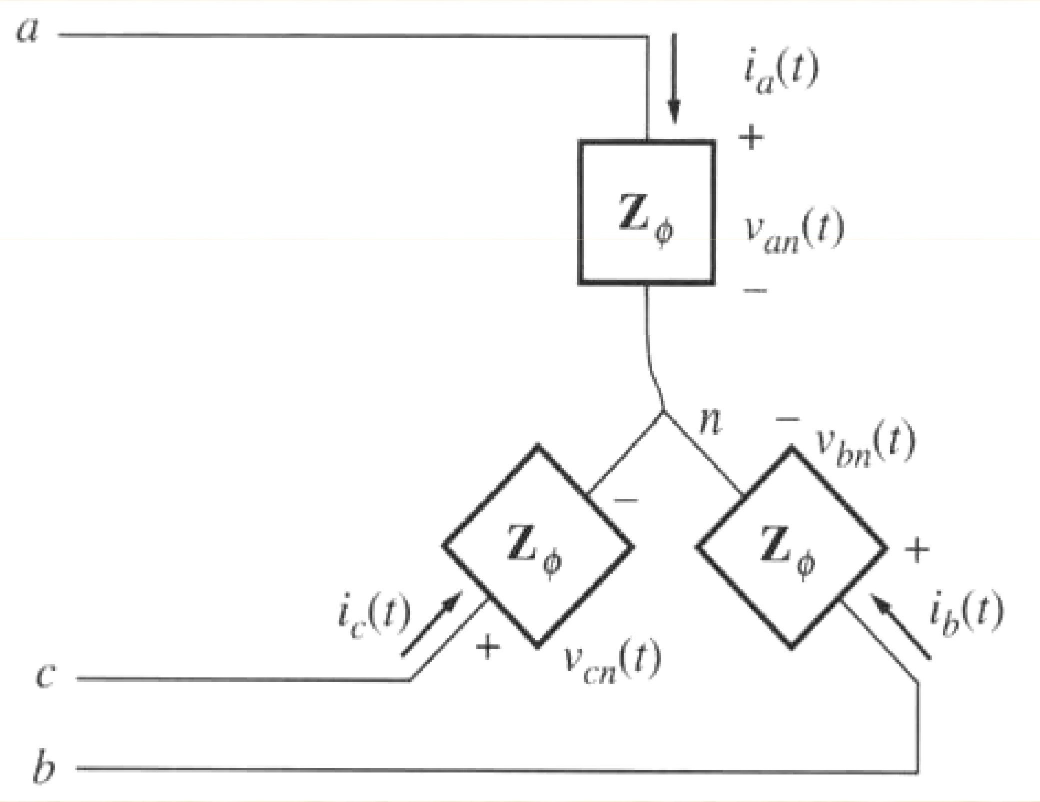


✓ **power in phases is Time Variant**

✓ **The total power supplied to the load is constant**

Power Relationships For a balanced Three-Phase load

- For a balanced Y-connected load with the impedance $Z_\phi = Z\angle\theta^o$:



Power Relationships For a balanced Three-Phase load

❑ **Using Phase quantities in each phase of a Y- or Δ-connection**

✓ Real Power: $P = 3V_\phi I_\phi \cos\theta = 3I_\phi^{\,2} Z\cos\theta$ $3I_\phi^2 R$

✓ Reactive Power: $Q = 3V_\phi I_\phi \sin\theta = 3I_\phi^{\,2} Z\sin\theta$ $3I_\phi^2 X$

✓ Apparent Power: $S = 3V_\phi I_\phi = 3I_\phi^{\,2} Z$

Power Relationships For a Balanced Three-Phase load

- **Using Line quantities of a Y-connected Load**

  - ✓ Real Power $$\boxed{P = 3V_{\phi} I_{\phi} \cos\theta}$$

  - Since for this load $I_L = I_{\phi}$ *and* $V_{\phi} = V_L / \sqrt{3}$

  - Therefore: $$P = 3\frac{V_L}{\sqrt{3}} I_L \cos\theta$$

  - Finally: $$\boxed{P = \sqrt{3} V_L I_L \cos\theta}$$

Power Relationships For a Balanced Three-Phase load

- **Using Line quantities of a Δ-connected Load**

  - ✓ Real Power $P = 3V_\phi I_\phi \cos\theta$

  - ➢ Since for this load $V_L = V_\phi \; and \; I_\phi = I_L / \sqrt{3}$

  - ➢ Therefore: $P = 3 \, V_L \dfrac{I_L}{\sqrt{3}} \cos \theta$

  - ➢ Finally: $P = \sqrt{3} V_L I_L \cos \theta$

Same as for a Y-connected load!

Power Relationships For a Balanced Three-Phase load

- **Using Line quantities of Y- or Δ-connection**
  - Reactive power: $$Q = \sqrt{3} V_L I_L \sin\theta$$
  - Apparent power: $$S = \sqrt{3} V_L I_L$$
- Note: θ is the angle between the phase voltage and the phase current – the impedance angle.

✓ **Power factor is:** $F_p = \cos\theta = P/S = P_\phi / S_\phi$